Fluids Quiz 1 Solution - Problem 1 S'08

$$z = 2h\frac{x}{c}\left(\frac{1}{2}-\frac{x}{c}\right), \quad \frac{dz}{dx} = 2\frac{h}{c}\left(\frac{1}{2}-2\frac{x}{c}\right) = 2\frac{h}{c}\left(\frac{1}{2}-(1-\cos\theta)\right) = 2\frac{h}{c}\left(\cos\theta-\frac{1}{2}\right)$$

a) $\boxed{A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}\,d\theta = \alpha - \frac{1}{\pi}\int_0^\pi 2\frac{h}{c}\left(\cos\theta-\frac{1}{2}\right)d\theta = \alpha + \frac{h}{c}}$

$\boxed{A_1 = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos\theta\,d\theta = \frac{2}{\pi}\int_0^\pi 2\frac{h}{c}\left(\cos^2\theta-\frac{1}{2}\cos\theta\right)d\theta = 2\frac{h}{c}}$

$\boxed{A_2 = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos 2\theta\,d\theta = \frac{2}{\pi}\int_0^\pi 2\frac{h}{c}\left(\cos\theta-\frac{1}{2}\right)\cos 2\theta\,d\theta = 0}$

$\boxed{A_3 = A_4 = \ldots = 0}$

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b) $\boxed{C_\ell = 2\pi\left[A_0 + \frac{1}{2}A_1\right] = 2\pi\left[\alpha + 2\frac{h}{c}\right]}$

$\boxed{C_{m_{c/4}} = \frac{\pi}{4}\left(A_2 - A_1\right) = -\frac{\pi}{2}\frac{h}{c}}$

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c) $C_\ell = 2\pi\left[\alpha - \alpha_{L=0}\right] = 2\pi\left[\alpha + 2\frac{h}{c}\right]$

$\boxed{\alpha_{L=0} = -2\frac{h}{c}}$

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